LINEAR MODEL EQUILIBRIUM

$$y_{t+1} = \alpha + \beta y_t + \epsilon_i \qquad E(\epsilon) = 0$$

ignore random variation

$$y_{t+1} = y_t$$

$$y_t - \beta y_t = \alpha$$

$$y_t(1-\beta) = \alpha$$

$$y_t = \frac{\alpha}{1-\beta}$$

MICROECONOMIC SOLUTION (SOLON 2004)

"PLUG AND PLAY"

$$C_{it} = (1-\tau) y_t - I_{it}$$

$$y_{it+1} = \mu + p[\theta \ln(I_{it} + G_{it}) + e_{it+1}]$$

NOT NEEDED

$$\delta + \lambda e_{it} + v_{it}$$

$$y_{it+1} = \mu + p\theta \ln(I_{it} + G_{it}) + p e_{it+1}$$

$$U = (1-\alpha)\ln\left[(1-t)y_t - I_{it}\right] + \alpha\mu + \alpha p\theta \ln(I_{it} + G_{it})$$

$$+ \alpha e_{it+1}$$

$$\frac{\partial U}{\partial I_{it}} =$$

$$\frac{(1-\alpha)}{(1-t)y_t - I_{it}} \cdot -1 + \alpha p \theta \frac{1}{I_{it} + G_{it}}$$

$$\frac{-(1-\alpha)}{(1-t)y_t - I_{it}} + \frac{\alpha p \theta}{I_{it} + G_{it}} = 0$$

$$\frac{\alpha p \theta}{I_{it} + G_{it}} = \frac{1-\alpha}{(1-t)y_t - I_{it}}$$

$$\frac{(1-t)y_t - I_{it}}{I_{it} + G_{it}} = \frac{1-\alpha}{\alpha p \theta}$$

$$\frac{I_{it} + G_{it}}{(1-t)y_t - I_{it}} = \cancel{\frac{1-\alpha}{\alpha p \theta}} \frac{\alpha p \theta}{1-\alpha}$$

$$I_{it} + G_{it} = \frac{\alpha p \theta (1-t) y_t}{1-\alpha} - \frac{\alpha p \theta I_{it}}{1-\alpha}$$

$$I_{it} = \frac{\alpha p \theta (1-t) y_t}{1-\alpha} - \frac{\alpha p \theta I_{it}}{1-\alpha} - G_{it}.$$

$$\times\frac{1-\alpha}{1-\alpha}$$

$$I_{it} + \frac{\alpha\rho\theta I_{it}}{1-\alpha} = \frac{\alpha\rho\theta(1-t)y_t}{1-\alpha} \overset{-}{\cancel{+}} G_{it}$$

$$\frac{I_{it}(1-\alpha) + \alpha\rho\theta I_{it}}{1-\alpha} = \frac{\alpha\rho\theta(1-t)y_t}{1-\alpha} \overset{-}{\cancel{+}} G_{it}$$

$$I_{it}(1-\alpha) + \alpha\rho\theta I_{it} = \alpha\rho\theta(1-t)y_t \overset{-}{\cancel{+}} G_{it}(1-\alpha)$$

$$I_{it}\left[(1-\alpha) + \alpha\rho\theta\right]$$

factor

$$1-\alpha+\alpha\rho\theta$$

$$1-\alpha(1+\rho\theta)$$

$$I^*_{it} = \frac{\alpha\rho\theta}{1-\alpha(1+\rho\theta)} y_t(1-t) \overset{-}{\cancel{+}} \frac{1-\alpha}{1-\alpha(1+\rho\theta)} G_{it}$$

~~SOLUTION FOR β~~ POSITIVE OR NEGATIVE?

$$\frac{\alpha\rho\theta}{1-\alpha(1+\rho\theta)} > 0$$

$$1-\alpha(1+\rho\theta) > 0$$

$$1 > \alpha(1+\rho\theta)$$

$$\frac{1}{1+\rho\theta} > \alpha$$

INTERGENERATIONAL MOBILITY

$$\ln y_{it+1} = \mu + p\, h_{it+1}$$

$$\downarrow$$

$$p\left[\theta \ln(I_{it} + G_{it}) + e_{it+1}\right]$$

Subsititute I^*_{it} for I_{it}

$$\cancel{\ln y_{it+1} = \mu + p\theta \ln\left[\frac{\alpha\theta p}{1-\alpha(1-\theta p)}(1-\tau) y_{it} \ldots\right] + p e_{it+1}}$$

$$\cancel{\frac{\alpha\theta p}{1-\alpha(1-\theta p)}(1-\tau) y_{it} - \frac{1-\alpha+1-\alpha(1-\theta p)}{1-\alpha(1-\theta p)} G_{it}}$$

$$\cancel{\frac{\alpha\theta p(1-\tau) y_{it} - 1-\alpha+1-\alpha(1-\theta p) G_{it}}{1-\alpha(1-\theta p)}}$$

$$\cancel{\frac{1-\alpha+1-\alpha+\alpha\theta p}{2-2\alpha+\alpha\theta p}}$$

$$\left[\frac{\alpha\theta p}{1-\alpha(1-\theta p)}\right][1-\tau]\, y_{it} - \left[\frac{1-\alpha}{1-\alpha(1-\theta p)}\right] G_{it} + G_{it}$$

NEGATIVE

$$\cancel{-1} \pm \cancel{\alpha} \pm 1 - \cancel{\alpha}(1-\theta p)$$

$$+ \left[\frac{\alpha\theta p}{1-\alpha(1-\theta p)}\right] G_{it}$$

FACTOR

$$\left[\frac{\alpha\theta p}{1-\alpha(1-\theta p)}\right]\left[(1-\tau) y_{it} + G_{it}\right]$$

FACTOR.. AGAIN

$$\left[\qquad\quad\right]\left[(1-\tau)y_{it} + \frac{G_{it}(1-\tau)y_{it}}{(1-\tau)y_{it}}\right]$$

$$\left[\frac{\alpha\theta\rho(1-\tau)}{1-\alpha(1-\theta\rho)}\right]\left[y_{it}\left(1+\frac{G_{it}}{(1-\tau)y_{it}}\right)\right]$$

$$\ln y_{it+1} = \mu + \theta\rho\ln\left[\frac{\alpha\theta\rho(1-\tau)}{1-\alpha(1-\theta\rho)}\right] + \theta\rho\ln\left[y_{it}\left(1+\frac{G_{it}}{(1-\tau)y_{it}}\right)\right] + \rho e_{it+1}$$

PROGRESSIVE GOVERNMENT INVESTMENT

$$\frac{G_{it}}{[(1-\tau)y_{it}]} \triangleq \varphi - \gamma\ln y_{it}$$

φ = phi

γ = gamma

γ = gamma

SUBSTITUTE BACK INTO EQUATION ABOVE

$$\ln y_{it+1} = \mu + \theta\rho\ln\left[\frac{\alpha\theta\rho(1-\tau)}{1-\alpha(1-\theta\rho)}\right] + \theta\rho\ln y_{it} + \theta\rho(\varphi - \gamma\ln y_{it}) + \rho e_{it+1}$$

$$\mu^{*} = \mu + \theta\rho\ln\left[\qquad\right] + \varphi\theta\rho$$

$$\theta\rho\ln y_{it} - \theta\rho\gamma\ln y_{it}$$

$$\theta\rho(1-\gamma)\ln y_{it}$$

$$\ln y_{it+1} \cong \mu^* + [(1-\gamma)\theta\rho] \ln y_{it} + \rho e_{it}$$